Temperature Modulation of Double Diffusive Convection in a Horizontal Fluid Layer

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Linear stability analysis is performed for the onset of thermosolutal convection in a horizontal fluid layer with rigid-rigid boundaries. The temperature field between the walls of the fluid layer consists of two parts: a steady part and a time-dependent periodic part that oscillates with time. Only infinitesimal disturbances are considered. The effect of temperature modulation on the onset of thermosolutal convection has been studied using the Galerkin method and Floquet theory. The critical Rayleigh number is calculated as a function of frequency and amplitude of modulation, Prandtl number, diffusivity ratio and solute Rayleigh number. Stabilizing and destabilizing effects of modulation on the onset of double diffusive convection have been obtained. The effects of the diffusivity ratio and solute Rayleigh number on the stability of the system are also discussed.

Key words: Double Diffusive Convection; Thermal Modulation; Rayleigh Number; Diffusivity Ratio.

1. Introduction

Natural convection generated by buoyancy due to simultaneous temperature and concentration gradients is generally referred to as ‘double diffusive convection’, ‘thermosolutal convection’, or ‘thermohaline convection’. Since double diffusive convection is related to many transport processes in nature and technology, it has received much attention over the past three decades. Although initial research in this area was in the field of oceanography [1], the study of double diffusive convection is of practical importance in a wide variety of fields involving convective heat and mass transfer, including astrophysics, geophysics, geology, chemistry and engineering [2]. The early work on this problem is summarized in several reviews [3 – 5].

In recent years crystal growth and casting of metallic alloys have stimulated studies in the solidification in binary systems. As an alloy solidifies, there is a rejection of one of the components into the melt, and so the resulting density difference between the two components together with the temperature gradient creates a double diffusive convection. Since the quality and structure of the resulting solid is influenced by the transport process in the fluid phase during the crystal growth, it should be of primary importance to have a good understanding of the double diffusive convection during solidification.

Stommel et al. [1] were the first to notice some properties of double diffusive convection with the discovery of the phenomenon of the salt fountain, which occurs when hot salty water lies above cold fresh water. Such a system was later analyzed by Stern [6], who noted the general properties of the motion now commonly known as ‘salt fingers’. The situation with reversed gradients (i.e. with the salt gradient stabilizing and the temperature gradient destabilizing) has been studied by Veronis [7], and stability criteria for horizontal boundaries of various kinds have been presented by Nield [8] by means of a linear stability analysis. Considering linear gradients, Baines and Gill [9] investigated the thermohaline convection in a fluid layer confined between two horizontal boundaries, which are dynamically free and conducting to both heat and salt. Chen [10] considered a two-dimensional problem of a linearly stratified salt solution contained between two infinite vertical plates and studied the onset of cellular convection due to a lateral temperature gradient. Proctor [11] studied the thermohaline convection in a horizontal fluid layer using rigid-rigid and free-free boundaries. Double diffusive convection in an inclined plane was investigated by Thangam et al. [12] for rigid-rigid boundaries.

Later on many other investigators studied this problem of double diffusive convection under various physical and boundary conditions. Sodha and Kumar [13] studied the stability of double diffusive convection in solar ponds with non-constant temperature and salinity gradients. Lopez et al. [14] have performed a linear stability analysis of triple diffusive convection in a horizontal fluid layer and found the effect of rigid-rigid boundaries on the onset of convection. Using linear stability analysis, Saunders et al. [15] studied the effect of gravity modulation on thermosolutal convection in an infinite layer of fluid using free-free boundaries. Gobin and Bennacer [16] investigated the problem of thermohaline convection in a vertical layer of a binary fluid and studied the onset of convection. Sezai and Mohamad [17] have performed a three-dimensional numerical study to investigate double diffusive, natural convection in a cubic enclosure subject to opposing and horizontal gradients of heat and solute imposed along the two vertical side walls. Most of these studies on double diffusive convection are mainly concerned with a uniform temperature gradient, which is independent of time.

However we can find many situations of practical importance in which the temperature gradient is a function of both space and time. This temperature gradient can be determined by solving the energy equation with suitable time-dependent thermal boundary conditions, and can be used as a mechanism to control the convective flow. Venezian [18] was the first who considered the above temperature gradient and studied the effect of temperature modulation on thermal instability in a horizontal fluid layer, nevertheless a similar problem has been studied earlier by Gershuni and Zhukhovitskii [19] for a temperature profile obeying a rectangular law. Some other researchers, who have also used this concept of temperature modulation in their studies of thermal instability in a horizontal fluid layer are: Rosenblat and Herbert [20], Rosenblat and Tanaka [21], Yih and Li [22], Roppo et al. [23], and Bhadauria and Bhatia [24]. Recently the author [25, 26] has investigated the effect of temperature modulation on the thermal instability in a horizontal fluid layer, and studied the effects of rotation and a vertical magnetic field. The above studies are performed using a single component fluid. However the literature on double diffusive convection with temperature modulation is scarce. Only recently Malashetty and Basavaraja [27] have investigated the effect of time-dependent boundary temperatures on the onset of double diffusive convection in a horizontal porous layer using free boundaries. To the best of the author's knowledge no other literature is available on double diffusive convection in which the temperature modulation effect has been considered in a fluid layer.

Therefore in the present study the effect of temperature modulation of rigid boundaries on double diffusive convection in a horizontal layer is investigated. A sinusoidal function is taken to modulate the walls' temperature. The results have been obtained for the following three cases: (a) when the plate temperatures are modulated in phase, (b) when the modulation is out of phase, and (c) when only the temperature of the lower plate is modulated, the upper plate being held at a fixed temperature.

Since the amplitude and frequency of modulation are externally controlled, the onset of double diffusive convection can be advanced or delayed by proper tuning of these parameters. Thus the temperature modulation can be used as an effective mechanism to control the quality and structure of the resulting solid by influencing the transport process during crystal growth.

2. Mathematical Formulation

We consider a Newtonian, incompressible binary fluid, confined between two parallel horizontal walls. Cartesian co-ordinates have been taken with the origin in the middle of the fluid layer and the z-axis vertically upwards, so that the fluid lies between the planes $z = -d/2$ and $z = d/2$. The walls are infinitely extended in x- and y-directions, and are rigid. A temperature gradient ΔT is maintained across the fluid layer by heating from below. Also we maintain a stabilizing uniform concentration gradient ΔS between the walls of the layer. The Soret and Dufour effects on heat and mass diffusion are assumed to be negligible. Then under the Boussinesq approximation the basic governing equations are

$$\frac{\partial \boldsymbol{V}}{\partial t} + \boldsymbol{V}\nabla\boldsymbol{V} = -\frac{1}{\rho_{\mathrm{R}}}\nabla p + \frac{\rho}{\rho_{\mathrm{R}}}\boldsymbol{g} + \nu\nabla^2\boldsymbol{V}, \quad (2.1)$$

$$\frac{\partial T}{\partial t} + \boldsymbol{V}\nabla T = \kappa_{\mathrm{T}}\nabla^2 T, \quad (2.2)$$

$$\frac{\partial S}{\partial t} + \boldsymbol{V}\nabla S = \kappa_{\mathrm{S}}\nabla^2 S, \quad (2.3)$$

$$\nabla\boldsymbol{V} = 0, \quad (2.4)$$

$$\rho = \rho_{\mathrm{R}}\left[1 - \alpha\,(T - T_{\mathrm{R}}) + \beta\,(S - S_{\mathrm{R}})\right], \quad (2.5)$$

where ρ_{R}, T_{R} and S_{R} are the constant reference density, temperature and concentration, respectively. $\boldsymbol{V} =$

(u,v,w) is the velocity, p the pressure, S the solute concentration, T the temperature, $\boldsymbol{g}=(0,0,-g)$ the acceleration due to gravity and t the time; ν is the kinematic viscosity, κ_T the thermal diffusivity, κ_S solute diffusivity, and α and β are the coefficients of thermal and solute expansion, respectively. For the temperature modulation of the boundaries we consider the following cases:

(i) When the temperature of the lower and upper boundary is modulated, we have

$$\begin{aligned}&T(t)=T_\mathrm{R}+\Delta T(1+\varepsilon\cos\omega t)\\ &\text{at } z=-d/2,\end{aligned} \tag{2.6a}$$

$$\begin{aligned}&T(t)=T_\mathrm{R}+\Delta T\varepsilon\cos(\omega t+\phi)\\ &\text{at } z=d/2.\end{aligned} \tag{2.6b}$$

(ii) When the upper boundary is held at a fixed constant temperature, then

$$\begin{aligned}&T(t)=T_\mathrm{R}+\Delta T(1+\varepsilon\cos\omega t)\\ &\text{at } z=-d/2,\end{aligned} \tag{2.7a}$$

$$T(t)=T_\mathrm{R} \text{ at } z=d/2. \tag{2.7b}$$

Here ΔT represents the temperature difference, ε is the amplitude of the modulation, ϕ the phase angle and ω the frequency of the modulation. Since we maintain a stabilizing uniform concentration gradient ΔS between the walls of the porous layer, the imposed boundary conditions on S are

$$\begin{aligned}&S=S_\mathrm{R}+\Delta S \text{ at } z=-d/2\\ &\text{and } S=S_\mathrm{R} \text{ at } z=d/2.\end{aligned} \tag{2.8}$$

2.1. Basic State

The basic state of the fluid is quiescent and can be given by

$$\begin{aligned}&\boldsymbol{V}=(u,v,w)=0,\quad T=T_\mathrm{b}(z,t),\quad S=S_\mathrm{b}(z),\\ &p=p_\mathrm{b}(z,t),\quad \rho=\rho_\mathrm{b}(z,t).\end{aligned} \tag{2.9}$$

The temperature $T_\mathrm{b}(z,t)$, concentration $S_\mathrm{b}(z)$, pressure p_b, and density ρ_b satisfy the equations

$$\frac{\partial T_\mathrm{b}}{\partial t}=\kappa_\mathrm{T}\frac{\partial^2 T_\mathrm{b}}{\partial z^2}, \tag{2.10}$$

$$\frac{\mathrm{d}^2 S_\mathrm{b}}{\mathrm{d}z^2}=0, \tag{2.11}$$

$$\frac{\partial p_\mathrm{b}}{\partial z}=-\rho_\mathrm{b} g, \tag{2.12}$$

$$\rho_\mathrm{b}=\rho_\mathrm{R}\left[1-\alpha(T_\mathrm{b}-T_\mathrm{R})+\beta(S_\mathrm{b}-S_\mathrm{R})\right]. \tag{2.13}$$

Equation (2.10) can be solved for the above cases (i) and (ii). We write

$$T_\mathrm{b}(z,t)=T_\mathrm{S}(z)+\varepsilon\mathrm{Re}\left\{T_0(z,t)\right\}, \tag{2.14}$$

where

$$T_\mathrm{S}(z)=T_\mathrm{R}+\Delta T\left(\frac{1}{2}-\frac{z}{d}\right), \tag{2.15}$$

$$T_0(z,t)=$$
$$\frac{\Delta T}{\sinh\lambda}\left\{\mathrm{e}^{\mathrm{i}\phi}\sinh\lambda\left(\frac{1}{2}+\frac{z}{d}\right)+\sinh\lambda\left(\frac{1}{2}-\frac{z}{d}\right)\right\}\mathrm{e}^{\mathrm{i}\omega t}, \tag{2.16}$$

and

$$\lambda^2=\mathrm{i}\omega d^2/\kappa_\mathrm{T}. \tag{2.17}$$

Solving (2.11) for concentrations with the boundary conditions (2.8), we get

$$S_\mathrm{b}=S_\mathrm{R}+\frac{\Delta S}{2}(1-2z/d). \tag{2.18}$$

2.2. Linear Stability Analysis

Let the system (2.9) be slightly perturbed. Then, in order to examine the behaviour of infinitesimal thermal disturbances to the basic state, we write

$$\begin{aligned}&\boldsymbol{V}=\boldsymbol{V'}=(u',v',w'),\quad T=T_\mathrm{b}(z,t)+\theta',\\ &p=p_\mathrm{b}(z,t)+p',\quad S=S_\mathrm{b}+S',\quad \rho=\rho_\mathrm{b}+\rho',\end{aligned} \tag{2.19}$$

where $\boldsymbol{V}$, θ', S', p' and ρ' represent the perturbed quantities which are assumed to be small. We substitute (2.19) into (2.1)–(2.4) and linearize with respect to the perturbation quantities $\boldsymbol{V'}$, θ', S' and p'. In order to non-dimensionalize the variables, we scale the length, time, temperature, velocity, pressure and modulation frequency according to

$$\begin{aligned}&\boldsymbol{r'}=d\boldsymbol{r^*},\quad t'=\frac{d^2}{\kappa_\mathrm{T}}t^*,\quad T_\mathrm{b}=\Delta T T_\mathrm{b}^*,\\ &\theta'=\Delta\theta^*,\quad \boldsymbol{V'}=\frac{\kappa_\mathrm{T}}{d}\boldsymbol{V^*},\quad S'=\Delta S S^*,\\ &p'=\frac{\rho_\mathrm{R}\kappa\nu}{d^2}p^*,\quad \omega=\frac{\kappa_\mathrm{T}}{d^2}\omega^*.\end{aligned} \tag{2.20}$$

Then the non-dimensionalized governing equations for the perturbed variables, namely the vertical components of the velocity w, temperature θ and solute concentration S, in linear form, are

$$Pr^{-1}\nabla^2\frac{\partial \mathrm{w}}{\partial \mathrm{t}} = \nabla^4 w + R_{\mathrm{a}}\nabla_1^2\theta - R_{\mathrm{S}}\nabla_1^2 S, \quad (2.21)$$

$$\frac{\partial\theta}{\partial \mathrm{t}} = -\left(\frac{\partial T_{\mathrm{b}}}{\partial z}\right)w + \nabla^2\theta, \quad (2.22)$$

$$\frac{\partial S}{\partial t} = -\left(\frac{\mathrm{d}S_{\mathrm{b}}}{\mathrm{d}z}\right)w + \tau\nabla^2 S, \quad (2.23)$$

where $\nabla_1^2 \equiv \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$ and $\nabla^2 \equiv \nabla_1^2 + \frac{\partial^2}{\partial z^2}$. The non-dimensionalized numbers which appear in the above equations are: the thermal Rayleigh number $R_{\mathrm{a}} = \frac{\alpha g \Delta T d^3}{\nu\kappa_{\mathrm{T}}}$, the solute Rayleigh number $R_{\mathrm{S}} = \frac{\alpha g \Delta S d^3}{\nu\kappa_{\mathrm{T}}}$, the Prandtl number $Pr = \nu/\kappa_{\mathrm{T}}$, and the diffusivity ratio $\tau = \kappa_{\mathrm{S}}/\kappa_{\mathrm{T}}$. The non-dimensional temperature and concentration gradients $\partial T_{\mathrm{b}}/\partial z$ and $\mathrm{d}S_{\mathrm{b}}/\mathrm{d}z$, which appear in (2.22) and (2.23), respectively, can be obtained from the dimensionless forms of (2.14) and (2.18) as

$$\frac{\partial T_{\mathrm{b}}}{\partial z} = -1 + \varepsilon \mathrm{Re}\left[g(z), \mathrm{e}^{\mathrm{i}\omega t}\right], \quad (2.24)$$

$$\frac{\mathrm{d}S_{\mathrm{b}}}{\mathrm{d}z} = -1, \quad (2.25)$$

where

$$g(z) = \frac{\lambda}{\sinh\lambda}\left\{\mathrm{e}^{\mathrm{i}\phi}\cosh\lambda\left(\frac{1}{2}+z\right) - \cosh\lambda\left(\frac{1}{2}-z\right)\right\} \quad (2.26)$$

and

$$\lambda^2 = \mathrm{i}\omega. \quad (2.27)$$

Now we seek the solution for the three unknown fields, namely velocity, temperature and concentration, using the normal mode technique as

$$\begin{pmatrix} w(x,y,z,t) \\ \theta(x,y,z,t) \\ S(x,y,z,t) \end{pmatrix} = \begin{pmatrix} w(z,t) \\ \theta(z,t) \\ S(z,t) \end{pmatrix} \exp[\mathrm{i}(a_x x + a_y y)]. \quad (2.28\text{a, b, c})$$

Substituting the expressions (2.28) in (2.21)–(2.23), we get

$$Pr^{-1}\left(D^2-a^2\right)\frac{\partial w}{\partial t} = \left(D^2-a^2\right)^2 w - a^2 R_{\mathrm{a}}\theta + a^2 R_{\mathrm{S}} S, \quad (2.29)$$

$$\frac{\partial\theta}{\partial t} = -\left(\frac{\partial T_{\mathrm{b}}}{\partial z}\right)w + \left(D^2-a^2\right)\theta, \quad (2.30)$$

$$\frac{\partial S}{\partial t} = -\left(\frac{\mathrm{d}S_{\mathrm{b}}}{\mathrm{d}z}\right)w + \tau\left(D^2-a^2\right)S, \quad (2.31)$$

where $a = \left(a_x^2 + a_y^2\right)^{1/2}$ is the horizontal wave number and $D \equiv \frac{\partial}{\partial z}$. The boundary conditions for the rigid and conducting walls are given by

$$w = Dw = \theta = S = 0 \quad \text{at} \quad z = \pm\frac{1}{2}. \quad (2.32)$$

3. Method

Using the Galerkin method, we transform the partial differential equations (2.29)–(2.31) into a system of ordinary differential equations. The ordinary differential equations are then solved numerically. We have

$$w(z,t) = \sum_{m=1}^{N} A_m(t)\psi_m(z), \quad (3.1)$$

$$\theta(z,t) = \sum_{m=1}^{N} B_m(t)\varphi_m(z), \quad (3.2)$$

$$S(z,t) = \sum_{m=1}^{N} C_m(t)\phi_m(z), \quad (3.3)$$

where

$$\psi_m(z) = \begin{cases} \dfrac{\cosh\mu_m z}{\cosh\dfrac{\mu_m}{2}} - \dfrac{\cos\mu_m z}{\cos\dfrac{\mu_m}{2}}, & \text{if } m \text{ is odd,} \\[2ex] \dfrac{\sinh\mu_m z}{\sinh\dfrac{\mu_m}{2}} - \dfrac{\sin\mu_m z}{\sin\dfrac{\mu_m}{2}}, & \text{if } m \text{ is even,} \end{cases} \quad (3.4)$$

$$\varphi_m(z) = \sqrt{2}\sin m\pi\left(z+\frac{1}{2}\right), \quad (3.5)$$

$$\phi_m(z) = \sqrt{2}\sin\left[(m+1)\pi z + (m-1)\frac{\pi}{2}\right] \quad (m = 1,2,3,\ldots,N). \quad (3.6)$$

The above functions $\psi_m(z)$, $\varphi_m(z)$ and $\phi_m(z)$ are defined such that each forms an orthonormal set in the interval $\left(-\frac{1}{2},\,\frac{1}{2}\right)$ and vanishes at $z=\pm\frac{1}{2}$. For the derivatives of $\psi_m(z)$ to vanish at $z=\pm\frac{1}{2}$, μ_m must be the roots of the characteristic equation [28]

$$\tanh\frac{1}{2}\mu_m-(-1)^m\tan\frac{1}{2}\mu_m=0. \tag{3.7}$$

Substituting the expressions (3.1)–(3.3) for w,θ and S in (2.29)–(2.31) and then multiplying by $\psi_n(z)$, $\varphi_n(z)$ and $\phi_n(z)$ $(n=1,2,3,\ldots,N)$, respectively, the resulting equations are then integrated with respect to z in the interval $\left(-\frac{1}{2},\frac{1}{2}\right)$. The outcome is a system of $3N$ ordinary differential equations for the unknown coefficients $A_n(t)$, $B_n(t)$ and $C_n(t)$ as given by

$$\begin{aligned}&Pr^{-1}\sum_{m=1}^{N}\left[K_{nm}-a^2\delta_{nm}\right]\frac{\mathrm{d}A_m}{\mathrm{d}t}=\\&\sum_{m=1}^{N}\left\{\left(\mu_m^4+a^4\right)\delta_{nm}-2a^2K_{nm}\right\}A_m\\&-a^2R_\mathrm{a}\sum_{m=1}^{N}P_{nm}B_m+a^2R_\mathrm{S}\sum_{m=1}^{N}U_{nm}C_m,\end{aligned} \tag{3.8}$$

$$\begin{aligned}\frac{\mathrm{d}B_n}{\mathrm{d}t}=\sum_{m=1}^{N}\left[P_{mn}-\varepsilon\mathrm{Re}\left\{F_{nm}\mathrm{e}^{\mathrm{i}\omega t}\right\}A_m\right]\\-\left(n^2\pi^2+a^2\right)B_n,\end{aligned} \tag{3.9}$$

$$\frac{\mathrm{d}C_n}{\mathrm{d}t}=\sum_{m=1}^{N}U_{mn}A_m-\tau\left[(n+1)^2\pi^2+a^2\right]C_n \quad (n=1,2,\ldots,N), \tag{3.10}$$

where δ_{nm} is the Kronecker delta. The other coefficients, which appear in (3.8)–(3.10), are given by

$$K_{nm}=\int_{-1/2}^{1/2}\psi_n(z)D^2\psi_m(z)\mathrm{d}z, \tag{3.11}$$

$$P_{nm}=\int_{-1/2}^{1/2}\psi_n(z)\varphi_m(z)\mathrm{d}z, \tag{3.12}$$

$$U_{nm}=\int_{-1/2}^{1/2}\psi_n(z)\phi_m(z)\mathrm{d}z, \tag{3.13}$$

and

$$F_{nm}=\int_{-1/2}^{1/2}\varphi_n(z)\psi_m(z)g(z)\mathrm{d}z. \tag{3.14}$$

$\tau=0.05$, $\varepsilon=0.0$			
S. No.	R_S	a_C	R_C
1.1	100.0	3.112	1717.7
1.2	500.0	3.104	1750.3
1.3	1000.0	3.10	1787.6

Table 1. Unmodulated case.

$R_\mathrm{S}=500.0$, $\varepsilon=0.0$			
S. No.	τ	a_C	R_C
2.1	0.03	3.10	1775.6
2.2	0.05	3.104	1750.3
2.3	0.5	3.113	1713.4

Table 2. Unmodulated case.

The above coefficients have been evaluated numerically ([29], p. 125). Now, for the computational purpose we introduce the notations

$$\begin{aligned}&x_1=A_1,\quad x_2=B_1,\quad x_3=C_1,\\&x_4=A_2,\quad x_5=B_2,\quad x_6=C_2\ldots,\end{aligned} \tag{3.15}$$

and rearrange (3.8)–(3.10) in the form

$$\frac{\mathrm{d}x_i}{\mathrm{d}t}=G_{ij}(t)x_j \quad (i,j=1,2,\ldots,3N), \tag{3.16}$$

where the coefficients $G_{ij}(t)$ are periodic in t with the period $2\pi/\omega$. The fundamental matrix $C=\lfloor x_{ij}(2\pi/\omega)\rfloor$ of the solutions has been obtained by integrating the system (3.16), using the Runge-Kutta-Gill procedure ([29], p. 217, 227). Eigenvalues of the matrix C are obtained using Rutishauser's method ([30], p. 116), and the stability of the solution of (3.16) is discussed with the help of the classical Floquet theory ([31], p. 55).

4. Results and Discussion

We did find during the process of numerical solution that it is sufficient to take $N=4$ (four Galerkin terms – two even and two odd), therefore all the following results have been obtained for $N=4$. The values of the critical Rayleigh number R_C and corresponding wave number a_C in the absence of modulation $(\varepsilon=0)$ are found as given in Tables 1 and 2.

Now we consider $\varepsilon\neq0$ and find the effect of the temperature modulation on double diffusive convection in a binary fluid layer. The results have been obtained by solving (3.16) for x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10}, x_{11} and x_{12}, i. e., a system of 12 simultaneous ordinary differential equations has been considered. The values of R_C have been calculated for the following three cases: (a) when the plate temperatures are modulated in phase, i. e., $\phi=0$; (b) when the plate temperatures are modulated out of phase, i. e.,

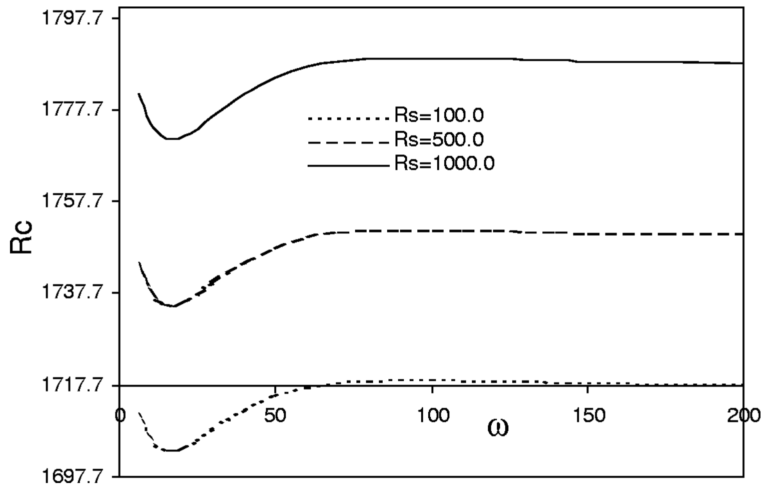

Fig. 1. In phase temperature modulation. Variation of R_C with ω; $\varepsilon = 0.4$; $Pr = 1.0$; $\tau = 0.05$.

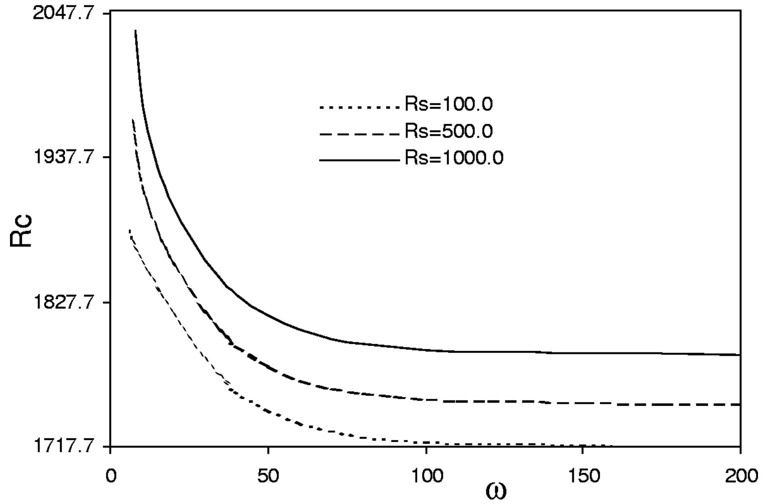

Fig. 2. Out phase temperature modulation. Variation of R_C with ω; $\varepsilon = 0.4$; $Pr = 1.0$; $\tau = 0.05$.

$\phi = \pi$; and (c) when only the bottom plate temperature is modulated, the upper plate being held at a fixed constant temperature, i. e., $\phi = i\infty$. The variation of the critical Rayleigh number R_C with respect to the modulation frequency ω and the amplitude of modulation ε, for different variables, are shown in Figures 1 – 8.

Figures 1 – 3 show the variation of R_C with ω for different values of the solute Rayleigh number R_S the values of the other parameters are $\varepsilon = 0.4$, $Pr = 1.0$, $\tau = 0.05$. We observe from the figures that the value of the critical Rayleigh number R_C increases with increase of the value of R_S, thus showing that the effect of increasing the solute Rayleigh number R_S is to delay the onset of double diffusive convection, as convection occurs at higher Rayleigh number.

In Figs. 4 – 6 we depict the variation of R_C with ω for different values of the diffusivity ratio τ, at $\varepsilon = 0.4$, $Pr = 1.0$, $R_S = 500.0$. From the figures we notice that on increasing the value of τ, the value of the critical Rayleigh number R_C decreases. Thus the effect of increasing the value of τ is to advance the onset of convection, as the onset of double diffusive convection takes place at a lower Rayleigh number.

Now, to find the effect of temperature modulation on the onset of double diffusive convection, first we consider case (a), i. e., in phase modulation. From Figs. 1 and 4 we observe that for small values of ω the effect of modulation is small, but destabilizing as convection occurs at a lower Rayleigh number than in the steady temperature gradient case (Tables 1 and 2). For intermediate values of ω the effect of modulation be-

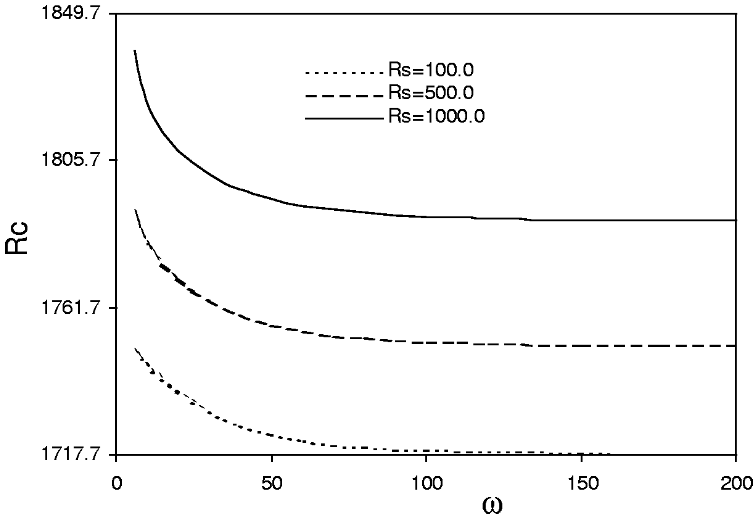

Fig. 3. Upper plate at constant temperature. Variation of R_C with ω; $\varepsilon = 0.4$; $Pr = 1.0$; $\tau = 0.05$.

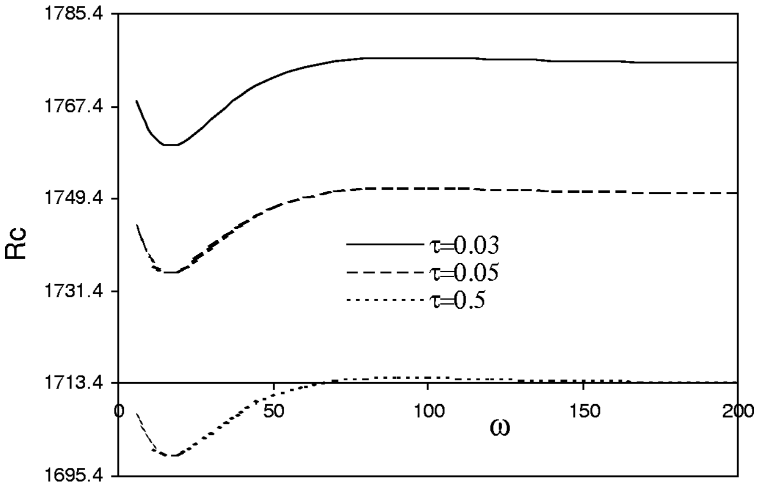

Fig. 4. In phase temperature modulation. Variation of R_C with ω; $\varepsilon = 0.4$; $Pr = 1.0$; $R_S = 500.0$.

comes maximal (destabilizing) near $\omega = 17$, and then decreases with increasing value of ω. It stabilizes the system at around $\omega = 60$, and finally falls off to zero as $\omega \to \infty$ (see the Tables 1 and 2). But when the temperature modulation is out of phase (Figs. 2 and 5), or when the upper plate is at constant temperature (Figs. 3 and 6), the effect of modulation is found to be stabilizing. The stabilizing effect is greatest near $\omega = 0$ and disappears altogether when the frequency ω becomes sufficiently large. We know that at high frequency, modulation becomes very fast, therefore the temperature in the fluid layer is unaffected by the modulation except for a thin layer, so that we find almost the same value of R_C as for zero modulation (Tables 1 and 2).

However, when the frequency of modulation is small, the effect of modulation is felt throughout the fluid layer. Further, the temperature profile consists of a steady straight line section plus a time-dependent parabolic part that oscillates with time. Now when the temperature modulation is in phase, this time-dependent parabolic profile becomes more and more significant as the amplitude of modulation increases. Since the parabolic profile is subject to finite amplitude instabilities the convection takes place at an early point thus destabilizing the system at low frequency. Further, when the modulation frequency increases, the effect of parabolic profile decreases, so the system becomes less destabilized and then at some frequency it becomes stabilized on further increasing the value of ω. But

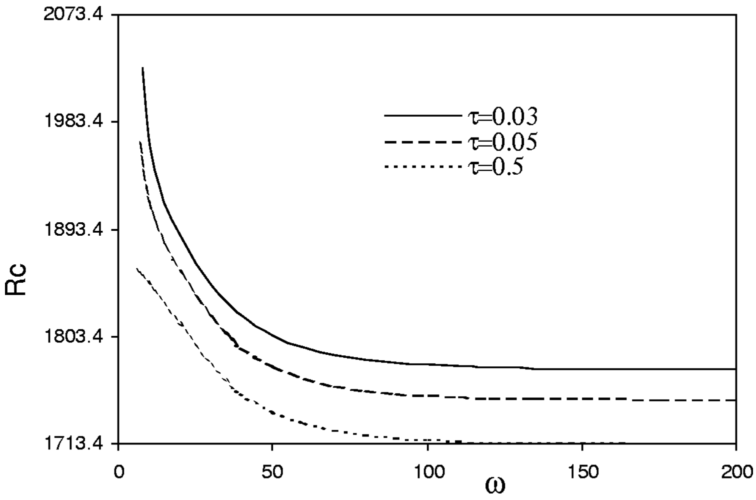

Fig. 5. Out phase temperature modulation. Variation of R_C with ω; $\varepsilon = 0.4$; $Pr = 1.0$; $R_S = 500.0$.

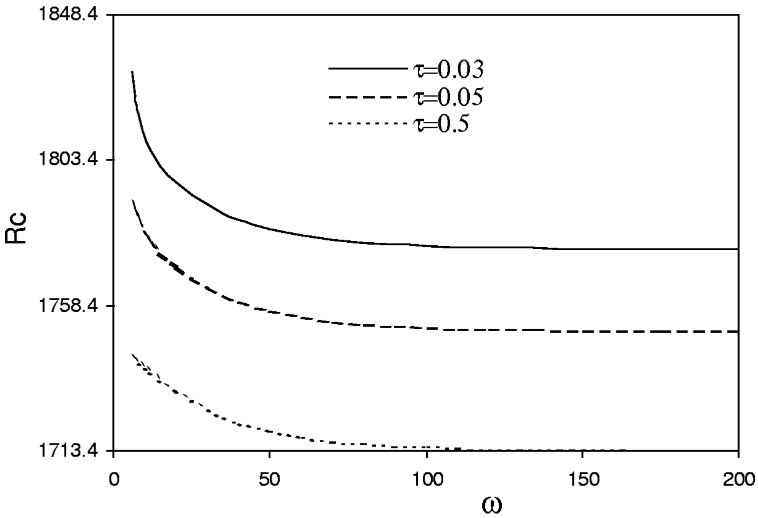

Fig. 6. Upper plate at constant temperature. Variation of R_C with ω; $\varepsilon = 0.4$; $Pr = 1.0$; $R_S = 500.0$.

when the temperature modulation is out of phase or the upper wall is at constant temperature, the convective wave propagates across the fluid layer, thereby inhibiting the instability, and so the convection occurs at higher Rayleigh number than that predicted by the linear theory with a steady temperature gradient. This propagation is greatest at low frequency, but decreases with increasing frequency; therefore the stabilizing effect is highest for small frequencies and decreases as the frequency increases.

Figure 7 depicts the variation of R_C with the amplitude of modulation ε, for all the three cases, at $\omega = 17.0$, $\tau = 0.05$, $R_S = 500.0$, $Pr = 1.0$. From the figure we find that for in phase modulation, R_C decreases when the amplitude of modulation ε increases. However for out of phase modulation, or when only the lower wall temperature is modulated, we observe that R_C increases as ε increases, thereby showing the stabilization of the system with increasing value of ε. In the last Fig. 8 we have compared the results corresponding to $N = 4$ and $N = 6$. It is found that the error in the results for $N = 4$ and $N = 6$ is about 0.074%. This justifies our calculations which correspond to $N = 4$ in this article.

5. Conclusion

In the present article we consider the effect of temperature modulation on double diffusive convection in a horizontal binary fluid layer with rigid-rigid bound-

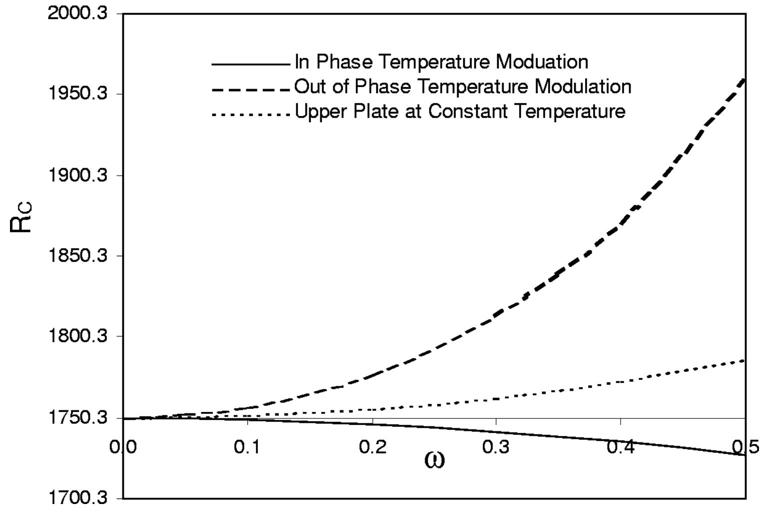

Fig. 7. Variation of R_C with ω; $\varepsilon = 0.4$; $Pr = 1.0$; $R_S = 500.0$; $\tau = 0.05$.

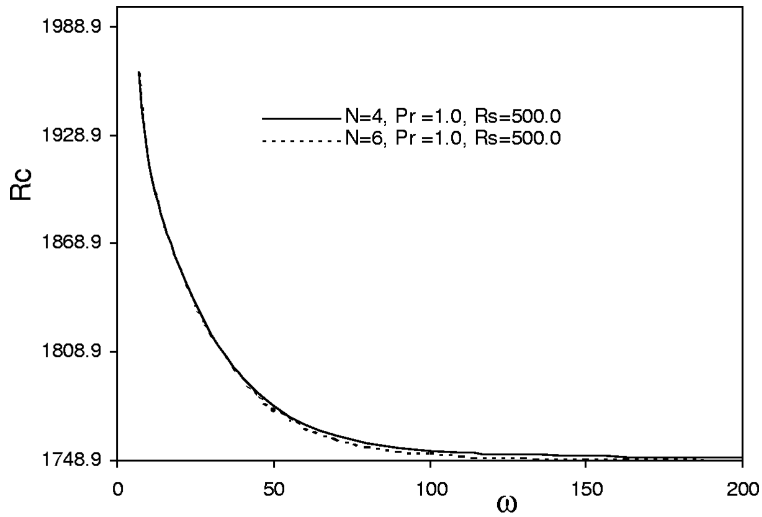

Fig. 8. Upper plate at constant temperature. Variation of R_C with ω; $\varepsilon = 0.4$; $\tau = 0.05$.

aries, under the assumptions that disturbances are infinitesimal and the amplitude of the applied temperature field is small. The following conclusions are drawn:

1.) The value of the critical Rayleigh number R_C increases on increasing the value of the solute Rayleigh number R_S. This shows that the effect of increasing the value of the solute Rayleigh number is to delay the onset of double diffusive convection.

2.) The effect of increasing the value of the diffusivity ratio is to decrease the value of the critical Rayleigh number, thus advancing the onset of double diffusive convection.

3.) We find that for in phase modulation, the modulation effect is small (destabilizing) when ω is small, becomes maximal (destabilizing) at around $\omega = 17$, decreases for intermediate values of ω, becomes stabilizing on further increasing the value of ω, and finally falls off to zero as $\omega \to \infty$.

4.) For the out of modulation case, or when only the lower wall temperature is modulated, the effect of modulation is found to be most stabilizing near $\omega = 0$, becomes less stabilizing for intermediate values of ω, and finally disappears as ω becomes very large.

5.) The applicability of the present theory, however, seems to be doubtful for the limit $\omega \to 0$ [18, 20].

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